# المراجعة رقم (1)

## اختبار شهر مارس

# Lesson (4) : Determinants

## second order

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

**Ex:1** Find the value of the following determinant :

a) $\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}$ b) $\begin{vmatrix} 4 & -7 \\ 2 & 6 \end{vmatrix}$ c) $\begin{vmatrix} 5 & 4 \\ -3 & -2 \end{vmatrix}$ d) $\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix}$

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- Third order

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & j \end{vmatrix} = a\begin{vmatrix} e & f \\ h & j \end{vmatrix} - b\begin{vmatrix} d & f \\ g & j \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= -a\begin{vmatrix} e & f \\ h & j \end{vmatrix} + b\begin{vmatrix} d & f \\ g & j \end{vmatrix} - c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

**Ex:2** Find the value of the following determinant :

a) $\begin{vmatrix} 4 & -1 & 3 \\ 0 & 5 & -2 \\ 0 & -3 & -1 \end{vmatrix}$ b) $\begin{vmatrix} -1 & 2 & 5 \\ 0 & 3 & -2 \\ 0 & 0 & 6 \end{vmatrix}$

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☐ Another method

$$\begin{vmatrix} a & b & c \\ d & e & l \\ m & n & k \end{vmatrix} \xrightarrow{\text{Repeat the first two}} \begin{vmatrix} a & b & c \\ d & e & l \\ m & n & k \end{vmatrix} \begin{matrix} a & b \\ d & e \\ m & n \end{matrix}$$

S1 = aek + blm + cdn
S2 = bdk + aln + cem
Then the value of the determinant is S = S1 – S2

➢ Remark :

(1) The triangular matrix:

It is a square matrix in which elements above or below principal diagonal are zeroes

Ex) $\begin{pmatrix} a & 0 \\ c & d \end{pmatrix}$, $\begin{pmatrix} a & b & c \\ 0 & e & l \\ 0 & 0 & k \end{pmatrix}$

Its determinant = $\begin{vmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{vmatrix} = a_{11} \times a_{22}$

And $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11} \times a_{22} \times a_{33}$

(2) Finding the area of triangle using determinants:

***If*** $\Delta ABC$ ***in which*** $A(x_1,y_1), B(x_2,y_2)$ ***and*** $C(x_3,y_3)$

Then the area of triangle ABC = $\frac{1}{2}|A|$ where A = $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Steps:

a) Find A = $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

b) Area = $\frac{1}{2}|A|$
Note: use elements of the 3rd column because it is easier

## (3) To prove that three points are collinear:

The three points $(x_1,y_1),(x_2,y_2)$ ***and*** $C(x_3,y_3)$ are collinear if

$$A = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \text{zero}$$

➢ *Cramer's rule*

### First: solving a system of linear equations of two variables:

To solve the two equations $ax + by = m$ and $cx + dy = n$ follow the steps:

1) Find the three determinants $\Delta$, $\Delta x$ and $\Delta y$ where

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}, \Delta x = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta y = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta \neq 0$$

2) To find the value of $x, y$

$$x = \frac{\Delta_x}{\Delta},\ y = \frac{\Delta_y}{\Delta}$$

*Note*: If $\Delta = 0$ then the system has no solution

### Second: solving a system of linear equations of three variables:

To solve the two equations $a_1x + b_1y + c_1z = m$ , $a_2x + b_2y + c_2z = n$ and $a_3x + b_3y + c_3z = k$ follow the steps:

1) Find the four determinants $\Delta$, $\Delta x$ , $\Delta y$ and $\Delta z$ where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta x = \begin{vmatrix} m & b_1 & c_1 \\ n & b_2 & c_2 \\ k & b_3 & c_3 \end{vmatrix}, \Delta y = \begin{vmatrix} a_1 & m & c_1 \\ a_2 & n & c_2 \\ a_3 & k & c_3 \end{vmatrix}$$

$$\Delta z = \begin{vmatrix} a_1 & b_1 & m \\ a_2 & b_2 & n \\ a_3 & b_3 & k \end{vmatrix}, \Delta \neq 0$$

2) To find the value of $x, y$ and $z$

$$x = \frac{\Delta_x}{\Delta},\ y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}$$

**Ex:3** solve the equation : $\begin{vmatrix} x & 0 & 1 \\ 8 & 1-x & -x \\ x & -1 & 1+x \end{vmatrix} = 0$

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**Ex:4** Find the area of a triangle whose vertices are

X(1,2) ,Y(3,-4) and Z(-2,3)

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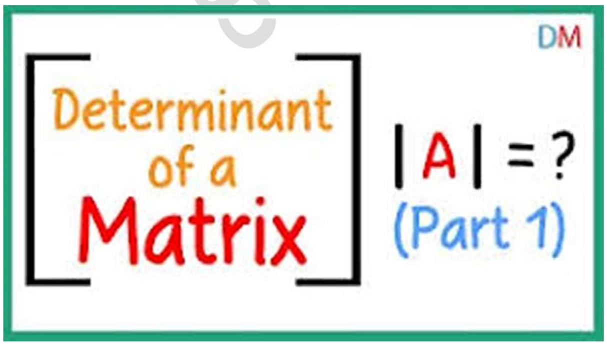

# Sheet 4

**1 Find the value of each of the following determinants :**

(1) $\begin{vmatrix} 7 & 5 \\ 3 & 2 \end{vmatrix}$ (2) $\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$ (3) $\begin{vmatrix} -2 & -2 \\ 4 & 0 \end{vmatrix}$

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**2 Prove that :**

(1) $\begin{vmatrix} 2x & -1 \\ 2 & 3x \end{vmatrix} + \begin{vmatrix} 3 & 6x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 13 \\ -2 & -7 \end{vmatrix}$

(2) $\begin{vmatrix} \csc\theta & \cot^2\theta \\ 1 & \csc\theta \end{vmatrix} \times \begin{vmatrix} 2 & -3 \\ 5 & -7 \end{vmatrix} = 1$

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**3 Find the value of each of the following determinants**

(1) $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 4 & 4 \\ 0 & 7 & 8 \end{vmatrix}$ (2) $\begin{vmatrix} 0 & 42 & 3 \\ 2 & 18 & 7 \\ 0 & 28 & 3 \end{vmatrix}$

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## 4 Solve each of the following equations

(1) $\begin{vmatrix} 2 & 1 \\ 4 & x \end{vmatrix} = 0$

(2) $\begin{vmatrix} x & -1 \\ 2 & x \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 2 & x \end{vmatrix} = 2$

(3) $\begin{vmatrix} 0 & -1 & x \\ x & 4 & 3 \\ 2 & 1 & 2 \end{vmatrix} = 10$

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**Find using determinants the area of the triangle :**

(1) A (2 , 4) , B (– 2 , 4) , C (0 , – 2)

(2) X (3 , 3) , Y (– 4 , 2) , Z (1 , – 4)

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**Use determinants to prove that each of the following points are collinear :**

(1) (3 , 5) , (4 , – 1) , (5 , – 7)

(2) (3 , 2) , (– 1 , 0) , (– 5 , – 2)

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**Solve each of the following systems of linear equations by Cramer's rule :**

(1) $2x - 3y = 5$ , $3x + 4y = -1$

(2) $x + 3y = 5$ , $2x + 5y = 8$

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**Solve each of the following systems of linear equations by Cramer's rule :**

(1) $2x + y - 2z = 10$ , $3x + 2y + 2z = 1$ , $5x + 4y + 3z = 4$

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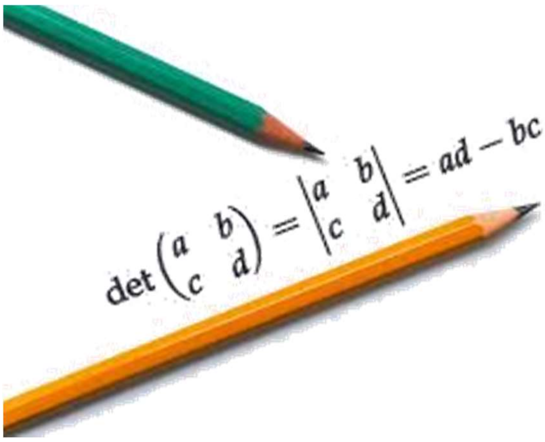

# Lesson (5) : Multiplicative inverse of a matrix

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Then $A^{-1} = \frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ $AA^{-1} = A^{-1}A = I$

$\Delta \neq 0$

1] Show the matrix which have multiplicative inverse :

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$

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d) $\begin{pmatrix} 2 & 6 \\ -1 & 3 \end{pmatrix}$ e) $\begin{pmatrix} -1 & 0 \\ 3 & 4 \end{pmatrix}$ f) $\begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$

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2] what is the real values of a which make each of the following matrices has A multiplicative inverse :

a) $\begin{pmatrix} a & 1 \\ 6 & 3 \end{pmatrix}$ b) $\begin{pmatrix} a & 9 \\ 4 & a \end{pmatrix}$

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3] if : X = $\begin{pmatrix} 1 & x \\ 0 & -x \end{pmatrix}$ prove that : $X^{-1} = X$

4] solve each of the following system using the matrices :

a) 3x+2y=5 , 2x+y=3

b) 2x-7y=3 , x-3y=2



# Sheet 5

**1 Show the matrices which have multiplicative inverse and the matrices which have not multiplicative inverse in the following , and find it if it is existed :**

(1) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ | (2) $\begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$ | (3) $\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$

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2] Find the real values of $x$ which make the matrix $\begin{pmatrix} x & 27 \\ 3 & x \end{pmatrix}$ have no multiplicative inverse.

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3] If $X = \begin{pmatrix} 1 & x \\ 0 & -1 \end{pmatrix}$ , **prove that :** $X^{-1} = X$

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4] If $A = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$ and $AB = \begin{pmatrix} 4 & -2 \\ 0 & 7 \end{pmatrix}$ , find the matrix B

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5] Solve each system of the following linear equations using the matrices :

(1) $3x + 2y = 5$ , $2x + y = 3$ | (2) $2x - 7y = 3$ , $x - 3y = 2$

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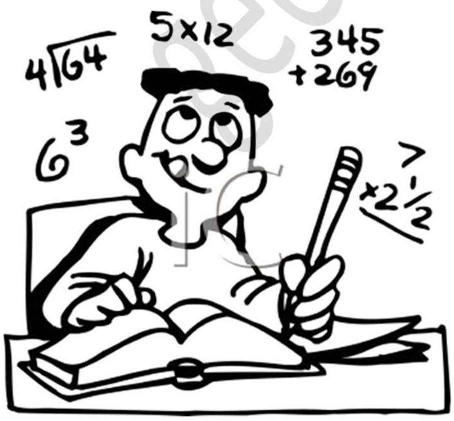
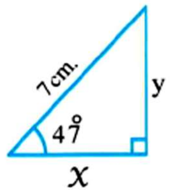


# Lesson (3)

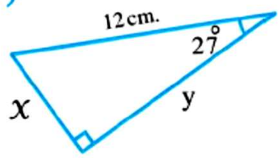
## Solving the right-angled triangle

**1** **Find the value of each of $x$ and y in each of the following figures :**

(1)




(2)

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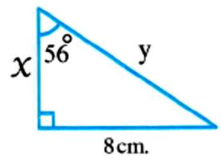


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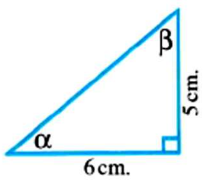
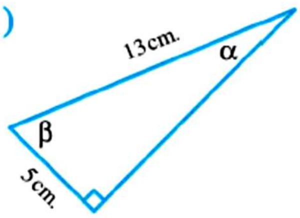
**2** **Find the value of each of the angles $\alpha$ and $\beta$ in degree measure in each of the following figures :**

(1)




(2)

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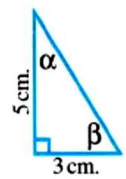


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**3** **ABC is a right-angled triangle at B. Find AB to one decimal , if :**

(1) m (∠ C) = $32° 18'$ and AC = 25 cm.

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# Sheet (3)

**1** **ABC is a right-angled triangle at B. Find AB to one decimal , if :**

(1) m (∠ C) = $54^\circ\ 13'$ and BC = 20 cm. « 27.7 cm. »

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**2** **ABC is a right-angled triangle at B. Find m (∠ C) to the nearest minute , if :**

(1) BC = 54 cm. and AC = 88 cm. « $52^\circ\ 9'$ »

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**3** **Solve the triangle ABC which is right-angled at B approximating the measures of angles to the nearest degree and the lengths of sides to the nearest cm. where :**

( 1 ) AB = 4 cm , BC = 6 cm. | ( 2 ) AB = 12.5 cm , BC = 17.6 cm.

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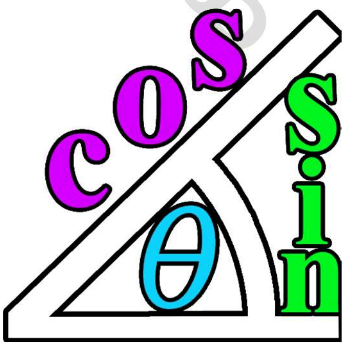


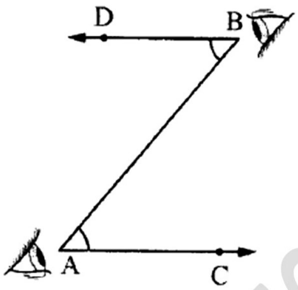
# Lesson (4)

## Angles of elevation and angles of depression

### Angle of elevation

If a person looked from the point A to an object at the point B above his horizontal sight , then the included angle between the horizontal ray $\overrightarrow{AC}$ and the seeing ray to above $\overrightarrow{AB}$ is called the elevation angle of B with respect to A

*i.e.* $\angle$ CAB is the elevation angle of B with respect to A




### Angle of depression

If a person looked from the point B to an object at the point A down his horizontal sight , then the included angle between the horizontal ray $\overrightarrow{BD}$ and the seeing ray to down $\overrightarrow{BA}$ is called the depression angle of A with respect to B

*i.e.* $\angle$ DBA is the depression angle of A with respect to B

## Sheet (4)

**1** From a point 8 metres apart from the base of a tree , it was found that the measure of the elevation angle of the top of the tree is 22°
Find the height of the tree to the nearest hundredth. « 3.23 m. »

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**2** A man found that the measure of the angle of elevation of the top of a tower , at a distance of 50 m. from its base , is 39° 21̀ Find the height of the tower. « 41 m. »

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**3** The length of the thread of a kite is 42 metres. If the measure of the angle which the thread makes with the horizontal ground equals 63° , find to the nearest metre the height of the kite from the surface of the ground. « 37 m. »

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**4** A person observed , from the top of a hill 2.56 km. high , a point on the ground. He found its depression angle measure was 63°. Find the distance between the point and the observer to the nearest metre. « 2873 m. »

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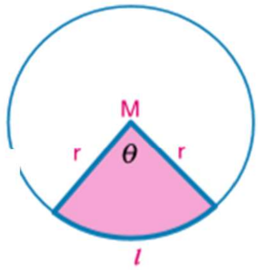
# Lesson (5)

## The Circular sector




**The circular sector:** is a part of the surface of the circle bounded by two radii and an arc .

Area of the circular sector $= \frac{1}{2} r^2 \theta^{rad}$ (where $\theta$ is the angle of the sector, r is the radius of the circle)

**Example**

1. Find the area of the circular sector in which the length of the radius of its circle is 10cm and the measure of its angle is $1.2^{rad}$

**Solution**

**Formula:** Area of the circular sector $= \frac{1}{2} r^2 \theta^{rad}$

**Substituting** $r = 10$, $\theta^{rad} = 1.2^{rad}$: $= \frac{1}{2} (10)^2 \times 1.2 = 60 \text{ cm}^2$

**Remember**

**Relation between the degree measure and the radian measure is:**

$$\frac{\theta^{rad}}{\pi} = \frac{x^\circ}{180^\circ}$$

**Example**

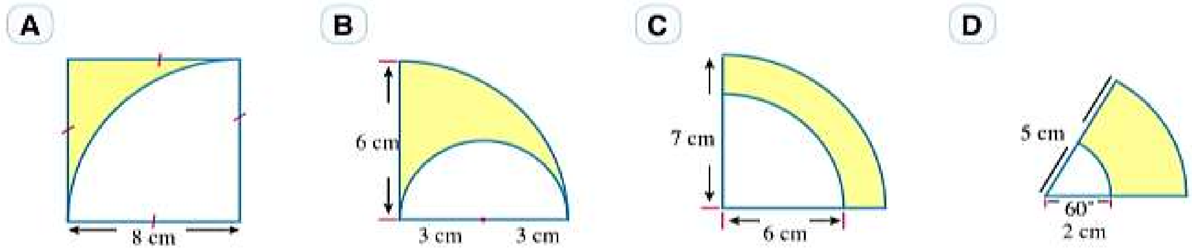
2. A circular sector in which the length of the radius of its circle equals 16cm, and the measure of its angle equals 120°, find its area to the nearest square centimetre .

**Solution**

**Formula:** area of the sector $= \frac{x^\circ}{360^\circ} \times \pi r^2$

**Substituting** $r = 16$, $x^\circ = 120^\circ$: $= \frac{120^\circ}{360^\circ} \times \pi (16)^2 \simeq 268 \text{ cm}^2$

1 Find in terms of $\pi$ the area of the shaded part in each of the following figures:

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**2** Find to the nearest $cm^2$ the area of a circular sector , where the measure of its central angle is $30°$ and the radius of its circle is of length 3.5 cm. « 3 $cm^2$ approximately »

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**3** Find the area of the circular sector in which the length of the radius of its circle is 10 cm. and the measure of its angle is $1.2^{rad}$ « 60 $cm^2$ »

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## Sheet (5)

**1** **Choose the correct answer from the given ones :**

( 1 ) The area of the circular sector = ……………

(a) $\frac{1}{2} \ell r^2$ (b) $\frac{1}{2} r \theta^{rad}$

(c) the area of the circle $\times \frac{\theta^{rad}}{2\pi}$ (d) the area of the circle $\times \frac{x^\circ}{180^\circ}$

( 2 ) The area of a sector whose arc is of length 10 cm. and the length of the diameter of its circle = 10 cm. equals ……………

(a) 50 $cm^2$ (b) 25 $cm^2$ (c) 12.5 $cm^2$ (d) 100 $cm^2$

( 3 ) The area of the circular sector in which the measure of its angle is $1.2^{rad}$ and the length of the radius of its circle is 4 cm. equals ……………

(a) 4.8 $cm^2$ (b) 9.6 $cm^2$ (c) 12.8 $cm^2$ (d) 19.6 $cm^2$

( 4 ) The perimeter of the circular sector in which the length of its arc is 4 cm. and the length of the diameter of its circle is 10 cm. equals ……………

(a) 14 cm. (b) 20 cm. (c) 30 cm. (d) 40 cm.

( 5 ) The area of the circular sector in which the measure of its angle is 120° , the length of the radius of its circle is 3 cm. equals ……………

(a) $3\pi$ $cm^2$ (b) $6\pi$ $cm^2$ (c) $9\pi$ $cm^2$ (d) $12\pi$ $cm^2$

( 6 ) The area of the circular sector in which , its perimeter is 12 cm. , length of its arc is 6 cm. equals ……………

(a) 6 $cm^2$ (b) 9 $cm^2$ (c) 12 $cm^2$ (d) 18 $cm^2$

( 7 ) If the perimeter of a sector is 8 cm. and its arc is of length 2 cm. , then its circle is of radius length ……………

(a) 6 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.

( 8 ) The arc of a sector is of length 3 cm. and the area of this sector is 15 $cm^2$ , then its circle radius is of length ……………

(a) 5 cm. (b) 10 cm. (c) 2.5 cm. (d) 15 cm.

( 9 ) The perimeter of a sector is 44 cm. Its circle is of radius length 14 cm. , then the length of the arc of the sector = ……………

(a) 16 cm. (b) 8 cm. (c) 32 cm. (d) 4 cm.

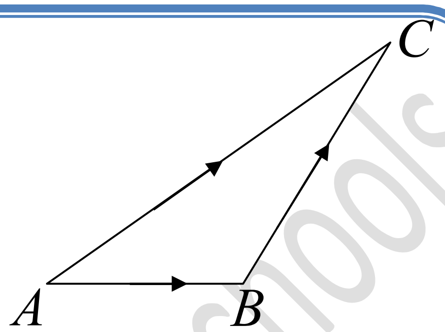
(10) If the area of the circular sector equals 110 $cm^2$ , the measure of its angle equals $2.2^{rad}$ , then the length of the radius of its circle equals ……………

(a) 2 cm. (b) 5 cm. (c) 10 cm. (d) 20 cm.

# Lesson (3)

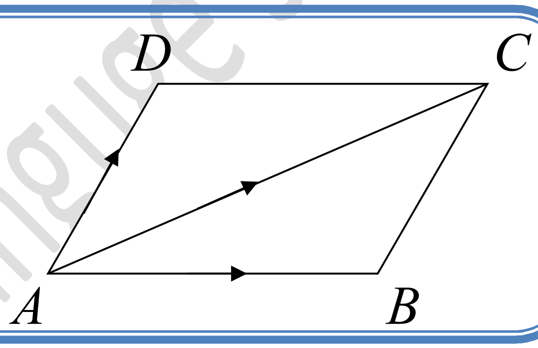
# Operation On Vectors

## First Adding vectors geometrically



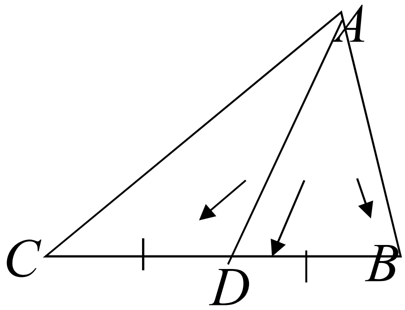

**1] the triangle rule :**

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$




**2] the parallelogram rule :**

$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$



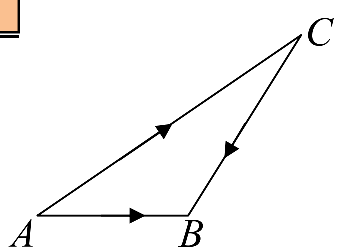

**3] the median rule:**

$\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$

## Second Subtracting two vectors geometrically

$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$

$\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A}$



**Example**

**In the quadrilateral ABCD , prove that :**

**(1)** $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD}$ | **(2)** $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{DC} + \overrightarrow{AD}$

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## Sheet (3)

### 1 Complete :

1. if : $\vec{A} = (-1,5)$ , $\vec{B} = (2,1)$ ,then $\|\overrightarrow{AB}\|$ = ……
2. if : $\vec{A} = (4,-2)$ , $\overrightarrow{AB} = (3,5)$ , then $\vec{B}$ = …..
3. if : M is a midpoint of $\overline{XY}$ ,then $\overrightarrow{XM} + \overrightarrow{YM}$ = ……
4. if : ABC is a triangle ,then $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$ = ….
5. if : ABC is a triangle ,then $\overrightarrow{AB} - \overrightarrow{CB}$ = ……. , $\overrightarrow{BA} - \overrightarrow{BC}$ = …….

### 2 ABCD is a trapezium in which in which $\overline{AD} // \overline{BC}$ , E is the midpoint of $\overline{AB}$

**F is the midpoint of $\overline{DC}$ .**

**prove that : $\overrightarrow{AD} + \overrightarrow{BC} = 2\,\overrightarrow{EF}$**

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### 3 ABCD is a quadrilateral in which : $\overrightarrow{BC} = 3\,\overrightarrow{AD}$ .prove that :

**a) ABCD is a trapezium** **b) $\overrightarrow{AC} + \overrightarrow{BD} = 4\,\overrightarrow{EF}$**

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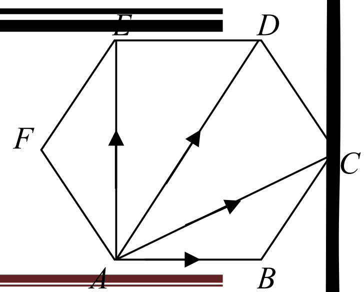
### 4 ABCDEF is regular hexagon prove that :




$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AE} + \overrightarrow{AF} = 2\,\overrightarrow{AD}$

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# Lesson (4)

# Application on Vectors

## First Geometric applications

We know that if $\overrightarrow{AB} = k\,\overrightarrow{DC}$ , $k \neq 0$ , then $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are :

- carried by the same straight line

**i.e.** : A , B , C , D are collinear.

or

- carried by two parallel straight lines

**i.e.** : $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$

**Remark**

If ABCD is a quadrilateral in which $\overrightarrow{AB} = k\,\overrightarrow{DC}$ , $k \neq 0$ , then $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ , $\|\overrightarrow{AB}\| = |k|\,\|\overrightarrow{DC}\|$ and vise versa.

**Example**

Use vectors to prove that : the points A (1, 4), B(–1, –2), C(2, –3) are vertices of right angled triangle at B.

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**Example**

Use the vectors to prove that: the points A (3, 4), B(1, –1), C(–4, –3), D(2, 2) are vertices of a rhombus.

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## Second Physical applications

### 1 The resultant force

- **The force :** is a vector passes through a given point and acts along a straight line.
- **The force :** is represented by a directed line segment and it is drawn by a suitable drawing scale.

**For example :**

1 A force of magnitude $F_1 = 10$ Newton acts in the East direction.

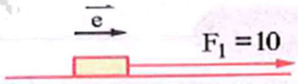
$\overrightarrow{F_1} = 10\,\vec{e}$

$\overrightarrow{F_1}$ is represented by a directed line segment of length 2 cm.

**Remember that :**

1 Consider $\vec{e}$ a unit vector in the East direction.

2 Choose a suitable drawing scale "Each 5 Newton is represented on drawing by 1 cm".




**Example**

If the forces: $\overrightarrow{F_1} = 2\,\vec{i} + \vec{j}$ , $\overrightarrow{F_2} = \vec{i} + 7\,\vec{j}$ , $\overrightarrow{F_3} = \vec{i} - 5\,\vec{j}$ act on a particle, Calculate the magnitude and direction of their resultant (forces are measured in Newton).

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## Relative Velocity

$$\overrightarrow{V_{BA}} = \overrightarrow{V_B} - \overrightarrow{V_A}$$

**Example**

A car (A) moves on a straight road with speed 70 km/h, A car (B) moves on the same road with speed 90 km/h. Find the relative velocity of car (A) with respect to car (B) when:

A The two cars move in the same direction.

B The two cars move in the opposite direction.

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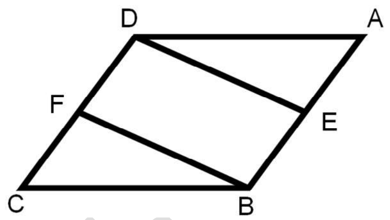
## Sheet (4)

**First** Geometry




1 ABCD **is a parallelogram** ,E **is a midpoint of** AB

F **is a midpoint of** DC

**Prove that** : DEBF **is a parallelogram**

2 ABCD **is a quadrilateral** , **if** $\overrightarrow{AC} + \overrightarrow{BD} = 2\,\overrightarrow{DC}$ **prove that :**

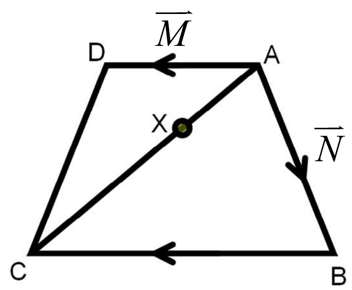
ABCD **is a parallelogram**

3 **using vectors prove that** : A(3,4) , B(1,-1) , C(-4,-3) , D(-2,2)

**are vertices of a rhombus**

4 **using vectors prove that** : A(1,3) , B(6, 1) , C(4,-4) , D(-1,-2)

**are vertices of a square and find its area**.




5 **ABCD is a trapezium** , AD//BC

$AD = \frac{1}{2} BC$ , $\overrightarrow{AB} = \overrightarrow{N}$ , $\overrightarrow{AD} = \overrightarrow{M}$

a) **Express in term of** $\overrightarrow{M}$ **and** $\overrightarrow{N}$ **each of the following** :

$\overrightarrow{BC}$ , $\overrightarrow{AC}$ , $\overrightarrow{DC}$ , $\overrightarrow{DB}$

b)**if** : $X \in \overrightarrow{AC}$ **where** $AX = \frac{1}{3} \times AC$

**prove that : the point** D , X **and** B **are collinear** .

## Second Physical application

### 1 Complete:

1. If: $\overrightarrow{F_1} = i - 3j$ , $\overrightarrow{F_2} = 3i - j$ act on a particle, then the norm of the resultant = ….N

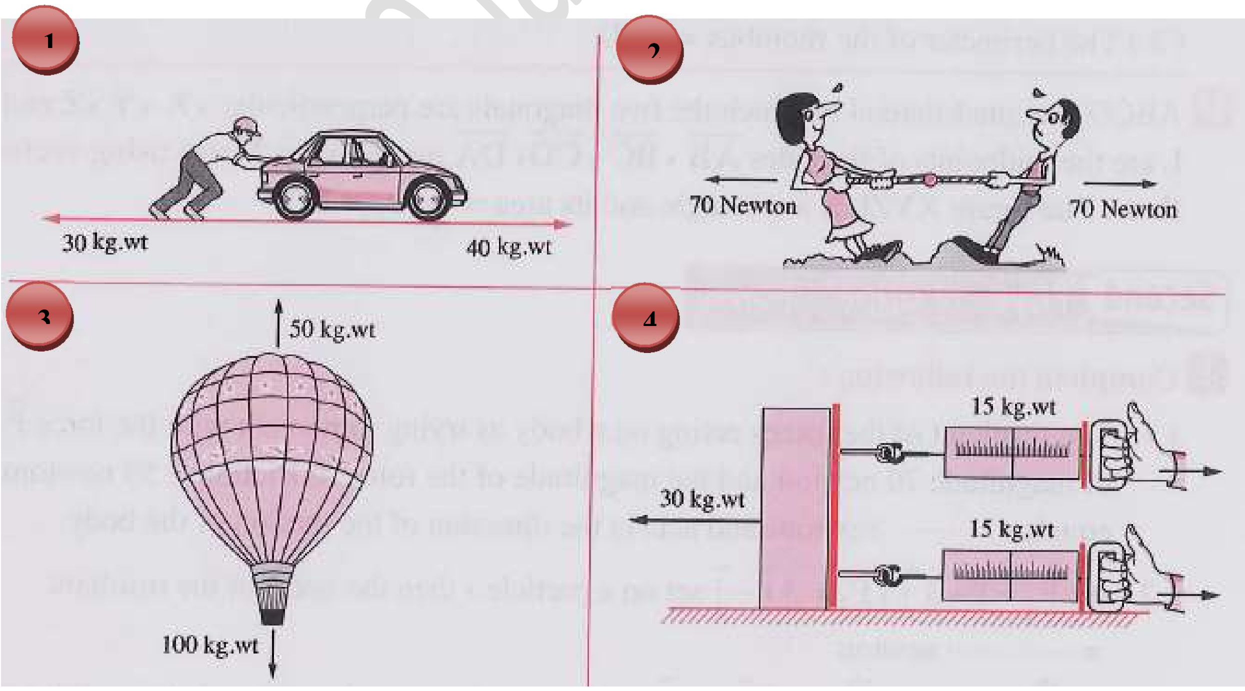
2. If: $\overrightarrow{F_1} = (a,b)$ , $\overrightarrow{F_2} = -3i + 4j$ act on a particle and the system is in equilibrium ,then a = …… , b = …..

3. If: $\overrightarrow{V_A} = 12\,\vec{e}$ , $\overrightarrow{V_B} = 8\,\vec{e}$ , then $\vec{V}_{AB}$ = ……

4. If : : $\overrightarrow{V_A} = 120\,\vec{e}$ , $\overrightarrow{V_B} = -80\,\vec{e}$ , then $\vec{V}_{BA}$ = …… , $\vec{V}_{AB}$ = ……

5. If: $\vec{V}_{AB} = 75\,\vec{e}$ , $\overrightarrow{V_A} = -60\,\vec{e}$ , then $\vec{V}_{BA}$ = …… , $\overrightarrow{V_B}$ = ……

### 2 Find the resultant force $\vec{F}$ acting in each of the following:



**3** <u>In each of the following, the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ act at a particle. Show the magnitude and the direction of the resultant of each two forces:</u>

1. $F_1 = 15$ newtons acts in the east direction,
   $F_2 = 40$ newtons acts in the west direction.
٢. $F_1 = 34$ gm.wt. acts in the north east direction,
   $F_2 = 34$ gm.wt. acts in the south west direction.
٣. $F_1 = 50$ dyne acts in $60°$ west of the north direction,
   $F_2 = 50$ dyne acts in $30°$ south of the east direction.
4. $F_1 = 30$ newtons acts in $20°$ east of the north direction ,
   $F_2 = 30$ newtons acts in $70°$ north of the east direction.

**4** Forces $\overrightarrow{F_1} = 7i - 5j$ , $\overrightarrow{F_2} = ai + 3j$ , $\overrightarrow{F_3} = -4i + (b-3)j$ ,find the values of $a$ and b if:

(1) The system of forces are in equilibrium.

(2) The resultant of the forces = -5j



# Lesson (1)

## Division of a line segment

### First: Finding the Coordinates of the point of division of a line segment by a certain ratio:

#### 1- Internal division

**If** $C \in \overrightarrow{AB}$ , then point C divides $\overrightarrow{AB}$ internally by the ratio $m_2 : m_1$

**where** $\frac{m_2}{m_1} > 0$ then $\frac{AC}{CB} = \frac{m_2}{m_1}$

and for the two directed segments $\overrightarrow{AC}$ , $\overrightarrow{CB}$

The same direction i.e.: $m_1 \times \overrightarrow{AC} = m_2 \times \overrightarrow{CB}$

**Let** $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x, y)$

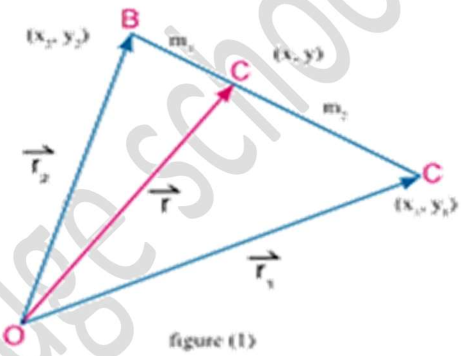



figure (1)

Then $\overrightarrow{r}\,(m_1 + m_2) = m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2}$

i.e.: $$\boxed{\overrightarrow{r} = \frac{m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2}}{m_1 + m_2}}$$ **which is called the vector form**

**Example**

(1) If A (2, − 1), B (− 3, 4), find the coordinates of point C which divides $\overrightarrow{AB}$ internally by the ratio 3 : 2 in the vector form.

**Solution**

Let C (x, y)

$\because A(2, -1)$ $\quad \therefore \overrightarrow{r_1} = (2, -1)$ , $\quad \because B(-3, 4)$ $\quad \therefore \overrightarrow{r_2} = (-3, 4)$

$m_2 : m_1 = 3 : 2$

$\because \overrightarrow{r} = \frac{m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2}}{m_1 + m_2}$

$\therefore \overrightarrow{r} = \frac{2(2, -1) + 3(-3, 4)}{2 + 3} = \frac{(4, -2) + (-9, 12)}{5} = \frac{(-5, 10)}{5} = (-1, 2)$

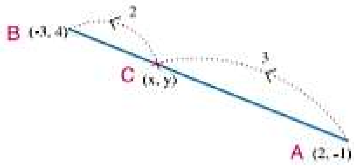
$\therefore$ The coordinates of point C are (−1, 2)

**Cartesian form:**

$$(x, y) = \frac{m_1(x_1, y_1) + m_2(x_2, y_2)}{m_1 + m_2} = \frac{(m_1 x_1 + m_2 x_2, m_1 y_1 + m_2 y_2)}{m_1 + m_2}$$

From that we get: $\boxed{(x\ ,\ y) = \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}\ ,\ \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}\right)}$

**Example**




2 Solve the previous example using the Cartesian form.

**Solution**

$$(x, y) = \left(\frac{2 \times 2 + 3 \times -3}{2 + 3}\ ,\ \frac{2 \times -1 + 3 \times 4}{2 + 3}\right) = (-1, 2)$$

## 2- External diviaion

If $C \in \overleftrightarrow{AB}$ ,$C \notin \overline{BA}$ , then C divides $\overrightarrow{AB}$ externally by the ratio $m_2 : m_1$ where $\frac{m_2}{m_1} < 0$ then one of the two values $m_1$ or $m_2$ is positive and the other is negative, then the following figure illustrates that there are two probabilities:

$C \in \overleftrightarrow{AB}$ , $C \notin \overline{AB}$

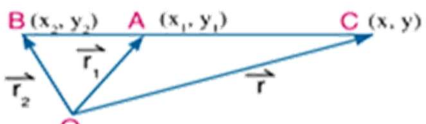



Figure (2)

$C \in \overleftrightarrow{AB}$ ,$C \notin \overline{AB}$

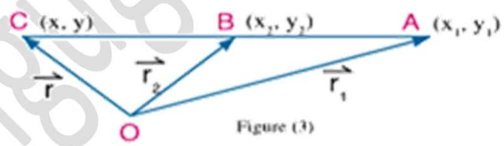
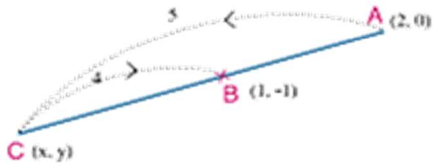



Figure (3)

**Example**

3 If A (2, 0), B (1, −1), find the coordinates of point C which divides $\overrightarrow{AB}$ externally by the ratio 5 : 4.

**Solution**




$\because \overrightarrow{r_1} = (2, 0), \overrightarrow{r_2} = (1, -1)$

$, m_2 : m_1 = 5 : -4 \quad \therefore \frac{m_2}{m_1} < 0$ negative — by substituting

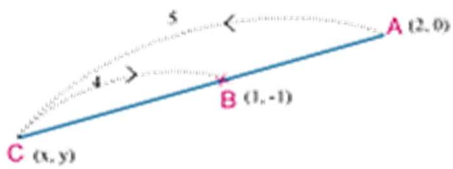
$, \overrightarrow{r} = \frac{m_1 \overrightarrow{r_1} + m_2 \overrightarrow{r_2}}{m_1 + m_2}$ — mathematical formula for the rule

$\therefore \overrightarrow{r} = \frac{-4(2, 0) + 5\,(1, -1)}{-4 + 5}$ — by distributing

$\overrightarrow{r} = (-8 + 5, 0 - 5) = (-3, -5)$ — by adding and simplifying

$\therefore$ The coordinates of point C are (−3, −5)

**Cartesian form:**




$$(x, y) = \left(\frac{-4 \times 2 + 5 \times 1}{-4 + 5}, \frac{-4 \times 0 + 5 \times -1}{-4 + 5}\right)$$

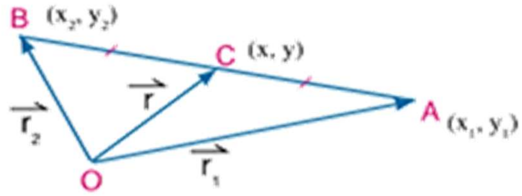
$$= (-3, -5)$$

**Notice that:**

If C is the midpoint of $\overrightarrow{AB}$ where A $(x_1, y_1)$, B$(x_2, y_2)$
then: $m_1 = m_2 = m$ then

$$\overrightarrow{r} = \frac{\overrightarrow{r_1} + \overrightarrow{r_2}}{2}$$ **Vector form**

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ **Cartesian form**




## Second : Finding the ratio of Division

If point C divides $\overrightarrow{AB}$ by the ratio $m_2 : m_1$ and:

**1-** The ratio of division $\frac{m_2}{m_1} > 0$ then the division is internal.

**2-** The ratio of division $\frac{m_2}{m_1} < 0$ then the division is external.

**Example**

**4** If A (5, 2), B (2, – 1) , find the ratio by which $\overrightarrow{AB}$ is divided by the points of intersection of $\overleftrightarrow{AB}$ with the two axes, showing the type of division in each case, then find the coordinates of the division point.

**Solution**

**First:** let the x–axis intersects $\overrightarrow{AB}$ at point C (x, 0)

where $\frac{AC}{CB} = \frac{m_2}{m_1}$ then: $y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

$\therefore 0 = \frac{m_1 (2) + m_2 (-1)}{m_1 + m_2} = \frac{2m_1 - m_2}{m_1 + m_2}$

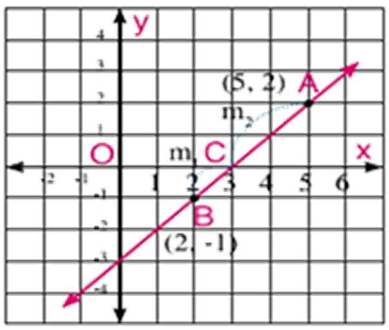
$\therefore 2m_1 = m_2$ $\therefore \frac{m_2}{m_1} = \frac{2}{1}$ (ratio of division)

$\because \frac{m_2}{m_1} > 0$

$\therefore$ The division is internal by the ratio 2 : 1

$\therefore$ The coordinates are C $\left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, 0\right) = \left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, 0\right)$

$= (3, 0)$




**Second:** The straight line intersects the y–axis at point D

Let the coordinates of D be (0, y)

where $\frac{AD}{DB} = \frac{m_2}{m_1}$ then $x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

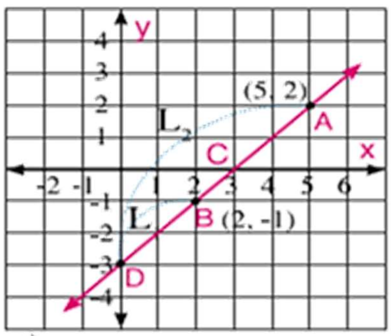
$\therefore 0 = \frac{m_1 \times 5 + m_2 \times 2}{m_1 + m_2}$

$\therefore 2\, m_2 = -5m_1$ $\therefore \frac{m_2}{m_1} = -\frac{5}{2}$ (ratio of division)

$\because \frac{m_2}{m_1} < 0$

$\therefore$ The division is external by the ratio 5 : 2
The coordinates of the point D are $(0, y) = \left(0\ , \frac{-2 \times 2 + 5 \times -1}{-2 + 5}\right)$

$\therefore (0, -3)$



## Sheet (1)

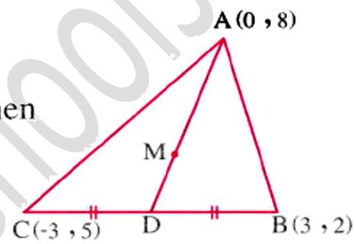
### 1 Complete the following :

( 1 ) If A = (3 , 6) , B = (– 7 , 4) , then the midpoint of $\overline{AB}$ = (…………… , ……………)

( 2 ) If M is the point of intersection of the two diagonals of the parallelogram ABCD where A = (3 , 7) , C = (– 3 , 1) , then M = (…………… , ……………)

( 3 ) If the point (3 , 6) is the midpoint of $\overline{AB}$ where A = (– 3 , 7) , then the point B = (…………… , ……………)

( 4 ) **In the opposite figure :**




$\overline{AD}$ is a median in $\Delta$ ABC , M is the point of intersection of its medians where A = (0 , 8) , B = (3 , 2) , C = (– 3 , 5) , then

the point D = (…………… , ……………)

the point M = (…………… , ……………)

**2** If A = (– 3 , – 7) , B = (4 , 0) , find the coordinates of the point C which divides $\overrightarrow{AB}$ by the ratio 5 : 2 internally. « (2 , – 2) »

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**3** If A = (0 , – 3) , B = (3 , 6) , find the coordinates of the point C which divides $\overrightarrow{BA}$ internally by the ratio 1 : 2 « (2 , 3) »

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**4** If A = (4 , 3) , B = (– 3 , 5) , find the point C $\in \overleftrightarrow{AB}$ where 3 AC = 5 CB

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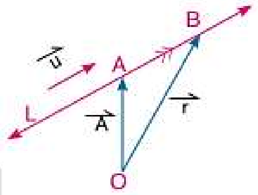
# Lesson (2)

## Equation of straight line

**Equaltion of the straight line given a point belonging to it and a direction vector to it**

**First:** Vector form $\overrightarrow{r} = \overrightarrow{A} + K\,\overrightarrow{u}$




**Example**

1. Write the vector equation of the straight line which passes through point (2, −3) and its direction vector is (1, 2).

**Solution**

Let the straight line pass through point A (2, −3) and $\overrightarrow{u} = (1, 2)$

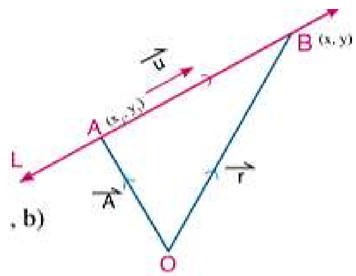
$\because\ \overrightarrow{r} = \overrightarrow{A} + K\,\overrightarrow{u}$ vector form of the equation of the straight line.

$\therefore$ The vector equation of the straight line is $\overrightarrow{r} = (2, -3) + K(1, 2)$.

**Second: The parametric equations**

The vector equation is $\overrightarrow{r} = \overrightarrow{A} + K\,\overrightarrow{u}$

$$x = x_1 + k a \quad , \quad y = y_1 + kb$$




**Third : Cartesian Equation**

Eliminating K from the parametric equations : $x = x_1 + ka$ , $y = y_1 + kb$

We get the equation: $\frac{x - x_1}{a} = \frac{y - y_1}{b}$ i.e.: $\frac{b}{a} = \frac{y - y_1}{x - x_1}$

Put $\frac{b}{a} = m$ (where m in the slope of the line), then the equation becomes in the form: $m = \frac{y - y_1}{x - x_1}$

**Example**

3. Find the Cartesian equation of the straight line which passes through the point (3,−4) and its direction vector is (2, −1)

**Solution**

$m = \frac{-1}{2}$ Slope of the line $m = \frac{b}{a}$

$m = \frac{y - y_1}{x - x_1}$ equation of the line given its slope and a point belonging to it.

$\frac{-1}{2} = \frac{y - (-4)}{x - 3}$ $m = \frac{-1}{2}$, $x_1 = 3$, $y_1 = -4$

$2y + 8 = -x + 3$ Product of means = product of extrems.

$x + 2y + 5 = 0$ general form.

# Sheet (2)

## Find the equation of the S.t line

1 Passing through (1 , 3) and its slope = $\frac{-2}{3}$

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2 Passing through the point (3 , -2) and its slope is -2

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3 Passing through the two points (3 , 1) and (5 , 4)

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4 Passing through the point (0 , -5) and makes with the positive direction of X – axis an angle of measure $135^0$.

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5 Passing through the point (-2 , 1) and parallel to the straight line

$\vec{r} = (2,-3) + k(1,0)$

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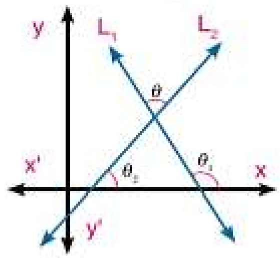
# Lesson (3)

## The angle between two

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \text{ where } m_1 m_2 \neq -1$$




1 Find the measure of the acute angle between the two straight lines whose equations are
$3x - 4y - 11 = 0$ , $x + 7y + 5 = 0$

**Solution**

A We find the slope of each straight line:

$m_1 = \frac{-3}{-4} = \frac{3}{4}$ slope of the first line

$m_2 = \frac{-1}{7}$ slope of the second line

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ Formula

$\tan\theta = \left|\frac{\frac{3}{4} - (-\frac{1}{7})}{1 + \frac{3}{4}(-\frac{1}{7})}\right|$ substituting the values of $m_1$, $m_2$

$= \left|\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{28}}\right| = \left|\frac{\frac{21+4}{28}}{\frac{28-3}{28}}\right| = 1$

$\theta = 45^\circ$

**Remember**
Slope of the straight line whose equation $ax + by + c = 0$ equals $\frac{-a}{b}$

## Sheet (3)

**1 Find the measure of the acute angle between the two straight lines whose slopes are :**

(1) $\frac{-3}{4}$ , $-7$　　(2) $\frac{1}{2}$ , $\frac{2}{9}$　　(3) $\frac{3}{4}$ , $-\frac{2}{3}$

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**2 Find the measure of the acute angle between each of the following pairs of straight lines :**

(1) $L_1 : \vec{r} = (0 , -2) + k (3 , -1)$ , $L_2 : \vec{r} = (0 , 5) + k (2 , 1)$

(2) $L_1 : \vec{r} = k (1 , 0)$ , $L_2 : \vec{r} = (3 , -2) + k (1 , -2)$

(3) $L_1 : \vec{r} = (0 , 1) + k (1 , 1)$ , $L_2 : 2x - y - 3 = 0$

(4) $L_1 : 2x + 3y = 15$ , $L_2 : \vec{r} = (-2 , -1) + k (1 , -3)$

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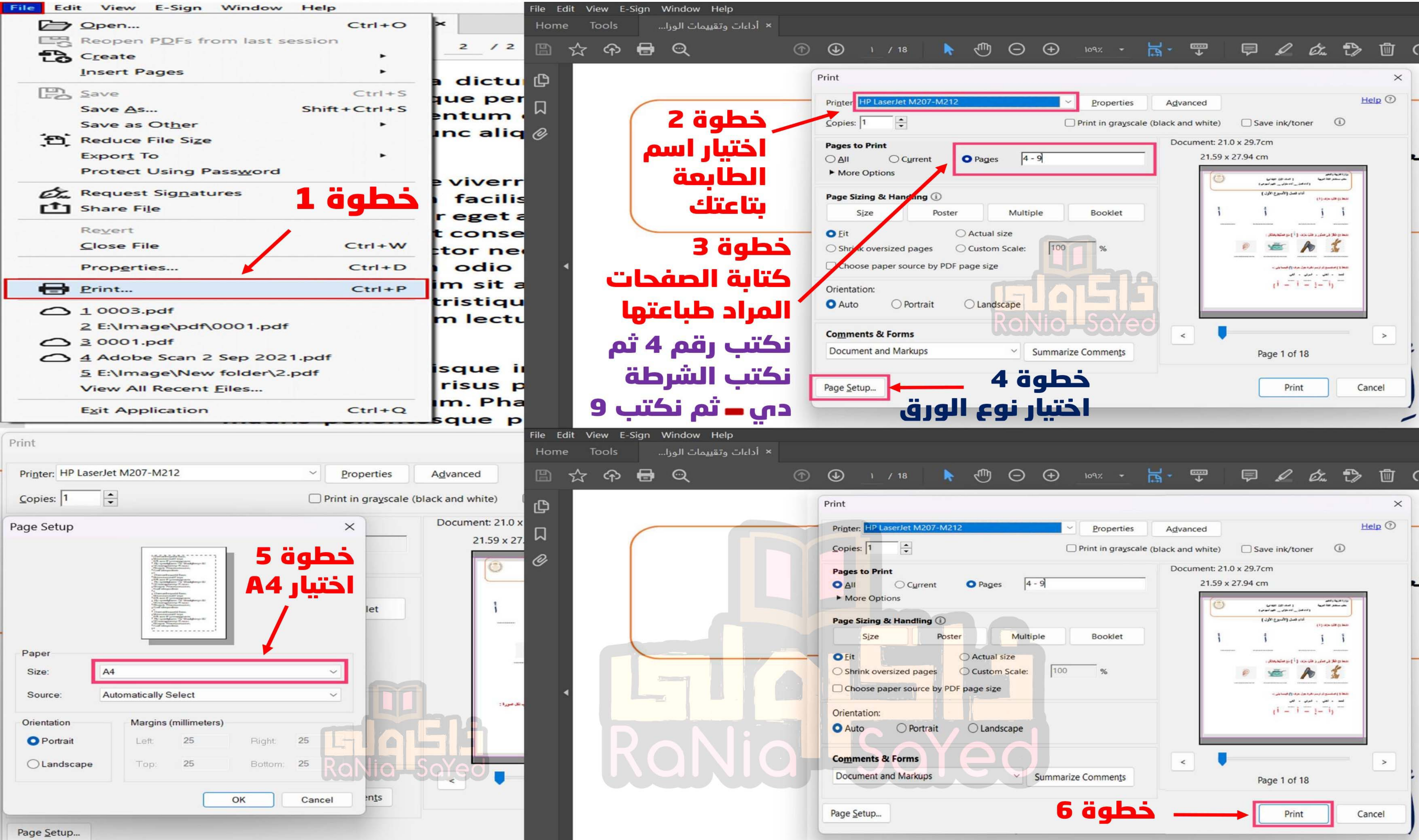
# كيفية طباعة صفحات معينة من ملف معين

## مثلا ازاى نطبع الصفحات من صفحة 4 الى صفحة 9



# المراجعة رقم (2)

## اختبار شهر مارس

# Basic concepts secondary one
# For April test

## First Algebra: -

- **Matrix Transpose:**
- $(A^t)^t = A$
- If A = $A^t$ then A is called symmetric matrix
- If A = $-A^t$ then A is called skew symmetric

**Transpose of product of two matrices**

$(AB)^t = B^t A^t$

## Determinants

- To find the value of the second order determinant

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

To find the value of $3^{rd}$ order determinant: $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

☐ **Applications on determinants**

**Finding the area of triangle using determinants:**

*If* $\triangle ABC$ *in which* $A(x_1,y_1)$, $B(x_2,y_2)$ *and* $C(x_3,y_3)$

Then the area of triangle ABC = $\frac{1}{2}|A|$ where A = $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

**To prove that three points are collinear:**

The three points $(x_1,y_1)$, $(x_2,y_2)$ *and* $C(x_3,y_3)$ are collinear if

A = $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ = zero

**Solving a system of linear equations (Cramer's rule)**

To solve the two equations $ax + by = m$ and $cx + dy = n$ follow the steps:

1) Find the three determinants $\Delta$, $\Delta x$ and $\Delta y$ where

$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, $\Delta x = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta y = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$,

2) To find the value of $x$, $y$ where $x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$ $\Delta \neq 0$

***Note***: If $\Delta = 0$ then the system has no solution

MULTIPLICATIVE INVERSE OF MATRIX

The multiplicative inverse of the matrix A is $A^{-1}$ where $AA^{-1} = I$ and this is not possible for all matrices

- The matrix A has a multiplicative inverse if $\Delta = |A| \neq 0$

To find the multiplicative inverse of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

**1)** Find $\Delta = |A| \neq 0$ 2) Then $A^{-1} = \frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

- **Second Geometry**
- The norm of the vector:

  It is the length of the line segment represents it

  If $\vec{A} = (x,y)$ then $\|\vec{A}\| = \sqrt{x^2 + y^2}$
- The unit vector: it is a vector whose norm is unity
- Zero vector: is denoted by $\vec{O} = (0 , 0)$ and its norm = zero and has no direction
  - Different forms of the vector

**(1)** Polar form

The polar form of $\vec{A} = (\|\vec{A}\| , \theta )$ where $\theta$ is the angle with positive direction of x-axis

**(2)** Cartesian form

$\overrightarrow{OA} = (x, y) = (\|\vec{A}\|\cos\theta , \|\vec{A}\|\sin\theta )$

Where $x = \|\vec{A}\|\cos\theta$ and $y = \|\vec{A}\|\sin\theta$

- Equivalent vectors

The two vectors are equivalent if they have the same norm and the same direction

Remark :

$\overrightarrow{AB} \neq \overrightarrow{BA}$ but $\overrightarrow{AB} = -\overrightarrow{BA}$

- The fundamental unit vector:

If $\vec{A} = (x , y)$ then $\vec{A} = x\vec{\imath} + y\vec{\jmath}$

## Parallel and perpendicular vectors

If $\vec{A} = (x_1 , y_1)$ and $\vec{B} = (x_2 , y_2)$ are two vectors then:

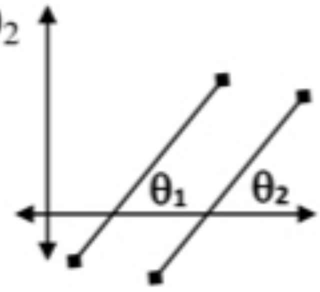
a) If $\vec{A} // \vec{B}$

Slope of $\vec{A}$ = slope of $\vec{B}$

$\tan\theta_1 = \tan\theta_2$

$\frac{y_1}{x_1} = \frac{y_2}{x_2}$

$x_1 y_2 = x_2 y_1$




$\boxed{x_1 y_2 - x_2 y_1 = 0}$

b) If $\vec{A} \perp \vec{B}$

Slope of $\vec{A}$ ×slopeof $\vec{B}$= -1

$\tan\theta_1 \times \tan\theta_2 = -1$

$\frac{y_1}{x_1} \times \frac{y_2}{x_2} = -1$

$x_1 x_2 = -y_1 y_2$

$\boxed{x_1 x_2 + y_1 y_2 = 0}$

Addition

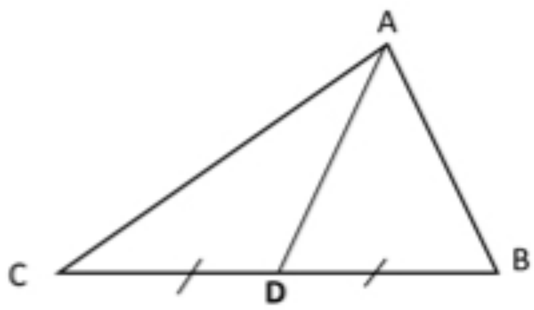
First: triangle rule

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\overrightarrow{AB} = -\overrightarrow{BA}$ where $\overrightarrow{AB} + \overrightarrow{BA} = \vec{0}$

from Parallelogram rule In $\Delta ABC$

if $\overline{AD}$ is a median then $\overrightarrow{AB} + \overrightarrow{AC} = 2\overrightarrow{AD}$




- The resultant force:

The resultant force of some forces $F_1$ , $F_2$ , $F_3$ , ……

Is $F = F_1 + F_2 + F_3 + ……$

<u>The relative velocity</u>

The relative velocity of A with respect of B is $\vec{V}_{AB}$ where $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

- If the two bodies A and B move in opposite direction, then:

$\vec{V}AB = \vec{V}_A - (-\vec{V}_B) = \vec{V}_A + \vec{V}_B$

## Division of line segment

The mid-point:

If A($x_1$ , $y_1$) and B($x_2$ , $y_2$) and C divides $\overrightarrow{AB}$ into two equal parts then

C = ( $\frac{x_1+x_2}{2}$ , $\frac{y_1+y_2}{2}$ )

To find the point of division

if the division Internally, If the division

internally externally:

$X_1 \quad m_1 \quad Y_1$ $\qquad$ $X_1 \quad m_1 \quad Y_1$

$+ \qquad\qquad\qquad\qquad -$

$X_2 \quad m_2 \quad Y_2$ $\qquad$ $X_2 \quad m_2 \quad Y_2$

## Equation of straight line

☐ The general form of the equation of straight line is $ax + by + c = 0$

☐ The slope of straight line m :

**a)** <u>From two points:</u> A($x_1$,$y_1$) , B($x_2$,$y_2$) $m = \frac{y_2-y_1}{x_2-x_1}$

**b)** <u>From the angle $\theta$ :</u> $m = tan\theta$

**c)** <u>From the equation :</u>

➢ If $x$ and $y$ in one side Then $m = \frac{-coefficient\ x}{coefficient\ y}$

The straight line whose equation is $y = mx$ pass through the origin point

- If $L_1 // L_2$ then $m_1 = m_2$
- If $L_1 \perp L_2$ then $m_1 \times m_2 = -1$

## ☐ Forming the equation of straight line

**1) Vector equation**: $\vec{r} = \vec{A} + k\vec{u}$ or $(x, y) = (x_1, y_1) + k(a , b)$

**2) The two Parametric equation**: from the vector form we can deduce the two parametric equations which are:

$x = x_1 + ka$ , $y = y_1 + kb$

**3) The Cartesian equation**: $y - y_1 = m(x - x_1)$ $\quad$ $where\ m = \frac{b}{a}$

And it is called the general equation of straight line.

➢ <u>Finding the equation of straight line given the intercepted parts of the two axes</u>

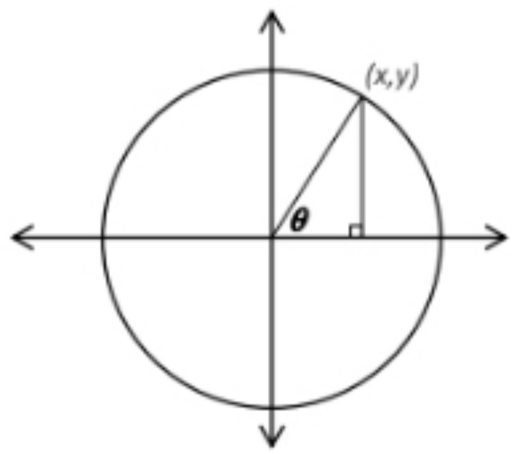
If the given is the two intersection points with x-axis and y-axis are $(a,0)$ and $(0,b)$ then the equation is: $\frac{x}{a}+\frac{y}{b}=1$

- **Third Trigonometry**
- **Trigonometric identity:**

☐ **Basic trigonometric identities**

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

☐ **From the unit circle:**




$x^2+y^2=1$ **then** $\sin^2\theta+\cos^2\theta=1$

Then : $\sin^2\theta = 1-\cos^2\theta$ $\cos^2\theta = 1-\sin^2\theta$

Dividing by $\cos^2\theta$

$\tan^2\theta+1=\sec^2\theta$

$\csc^2\theta = 1+\cot^2\theta$

☐ **finding the general solution**

**Steps:**

a) **Determine the quadrant**

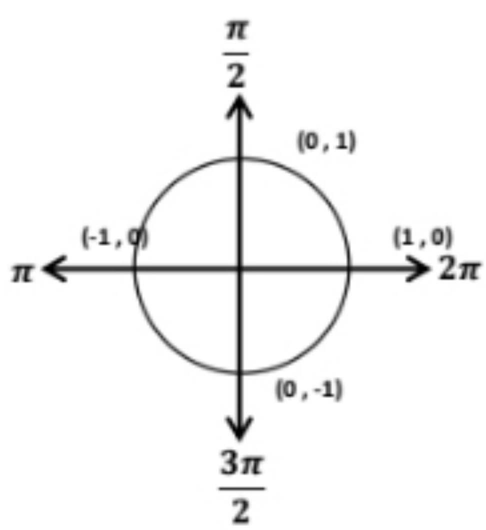
b) **Find the angle " shift …… "**

c) **Add $2\pi n$ in case of sin and cos**

**Add $\pi n$ in case of tan and cot**

**Very important remarks**

$$\left.\begin{array}{l}\sin\alpha=\cos\beta\\ \sec\alpha=\csc\beta\\ \sec\alpha=\csc\beta\end{array}\right\}\ \alpha+\beta=90$$




**And for the general solution:**

$$\left.\begin{array}{l}\sin\alpha=\cos\beta\\ \sec\alpha=\csc\beta\end{array}\right\}\ \alpha\pm\beta=90+2\pi n$$



$\alpha+\beta=90+\pi n \longrightarrow \tan\alpha = cot$

First Sec. 1
April

MATHEMATICS

# REVISION

1- $(X^t)^t - X = \dots\dots$

O (b) $X$ (c) $2X$ (d) zero

2- If $3^{\sin\theta} = 1$, where $\theta \in ]0, 2\pi[$, then $\theta = \dots\dots^\circ$

(a) 45 (b) 90 180 (d) 270

3- If $X + \begin{pmatrix} 3 & -2 \\ 5 & 0 \end{pmatrix} = O$, then $X = \dots\dots$

$\begin{pmatrix} -3 & 2 \\ -5 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 5 \\ -2 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 2 \\ -5 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} -3 & -2 \\ -5 & 0 \end{pmatrix}$

4- The value of the determinant $\begin{vmatrix} 4 & 0 & 0 \\ 7 & 2 & 0 \\ 1 & 5 & 1 \end{vmatrix} = \dots\dots$

(a) 1 (b) 2 (c) 4 8

5- If $\overrightarrow{X} = \overrightarrow{O}$, $\overrightarrow{X} = (a-3, b+5)$, then $a + b = \dots\dots$

$-2$ (b) 2 (c) 8 (d) 15

6- If $k \|4\overrightarrow{A}\| = \|-3\overrightarrow{A}\|$, then $k = \dots\dots$

$\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 1 (d) 12

7- In the Cartesian coordinates plane, if $\overrightarrow{OA} = (6, 6\sqrt{3})$, then its polar form is $\dots\dots$

$(12, \frac{\pi}{3})$ (b) $(12, \frac{\pi}{6})$ (c) $(6, \frac{\pi}{3})$ (d) $(6, \frac{\pi}{6})$

8- If $\overrightarrow{A} = (3, 4)$, $\overrightarrow{B} = (k, -8)$ and $\overrightarrow{A} \parallel \overrightarrow{B}$, then $k = \dots\dots$

$-6$ (b) 3 (c) 4 (d) 6

9- The matrix $(3 \quad 2 \quad 1)$ is of order $\dots\dots$

(a) $2 \times 1$ (b) $3 \times 2$ $1 \times 3$ (d) $3 \times 1$

10- If $\overrightarrow{C} = (5, 1)$, $\overrightarrow{D} = (-2, 4)$, then $\|\overrightarrow{C} + \frac{1}{2}\overrightarrow{D}\| = \dots\dots$

(a) 25 (b) 7 5 (d) 1

## MATHEMATICS

**11-** $\overrightarrow{AB} - \overrightarrow{BA} = \dots\dots$

(a) zero ● $2\,\overrightarrow{AB}$ (c) $2\,\overrightarrow{BA}$ (d) $\overrightarrow{O}$

**12-** The value of the expression : $5\cos\theta \times 3\sec\theta = \dots\dots$

(a) 1 (b) 2 (c) 8 ● 15

**13-** If the product of the two matrices $A \times B = I$ and the matrix $A = \begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}$, then the matrix $B = \dots\dots$

(a) $\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}$ (b) $\begin{pmatrix} 8 & 5 \\ 3 & 2 \end{pmatrix}$ ● $\begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} -2 & 5 \\ 3 & -8 \end{pmatrix}$

**14-** If $\overrightarrow{A} = (3, 5)$ , $\overrightarrow{B} = (4, 6)$ , then $\|-2\,\overrightarrow{A} + 3\,\overrightarrow{B}\| = \dots\dots$

(a) 6 (b) 8 ● 10 (d) 14

**15-** The slope of the striaght line perpendicular to the line with equation : $\vec{r} = (-3, 4) + k\,(2, 5)$ equals $\dots\dots$

(a) $\frac{-4}{3}$ ● $\frac{-2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{2}$

**16-** The simplest form of the expression : $\frac{\cot^2\theta - \csc^2\theta}{\cot\theta}$ is $\dots\dots$

(a) $-1$ (b) $\cot\theta - \csc^2\theta$ (c) $-\cot\theta$ ● $-\tan\theta$

**17-** If $\begin{vmatrix} 2 & 0 & 0 \\ 4 & x & 0 \\ -3 & 36 & x \end{vmatrix} = 50$ , then $x = \dots\dots$

(a) $\pm 6$ ● $\pm 5$ (c) 6 (d) 25

**18-** The value of $x$ which makes the matrix $\begin{pmatrix} 6 & 2 \\ x-4 & -4 \end{pmatrix}$ has no multiplicative inverse is $\dots\dots$

● $-8$ (b) $-10$ (c) 8 (d) 10

**19-** If $\overrightarrow{AB} = (3, -4)$ , $\overrightarrow{BC} = (2, 1)$ , then $\overrightarrow{CA} = \dots\dots$

(a) $(1, -5)$ (b) $(5, -3)$ (c) $(-3, 5)$ ● $(-5, 3)$

**20-** The ratio that the $x$-axis divides $\overline{BA}$ where $A\,(3, 2)$ , $B\,(5, 6)$ equals $\dots\dots$

(a) 3 : 5 internally. (b) 5 : 3 externally. (c) 1 : 3 internally. ● 3 : 1 externally.

First Sec. 1
April

**MATHEMATICS**

| | |
|---|---|
| **21-** | If $\overrightarrow{A} = (-9, 3)$ , $\overrightarrow{B} = (-2, 27)$ , then $\lVert \overrightarrow{AB} \rVert =$ ............... <br> (a) 13    (b) 15    (c) 20    25 |
| **22-** | The point which divides $\overline{AB}$ where A $(5, -6)$ , B $(-1, 3)$ with ratio $1 : 2$ internally is ............... <br> (a) $(0, 0)$    (b) $(3, 3)$    (c) $(-3, -3)$    $(3, -3)$ |
| **23-** | If $\overrightarrow{v_A} = 70\,\hat{i}$ , $\overrightarrow{v_B} = -20\,\hat{i}$ , then $\overrightarrow{v_{AB}} =$ ............... $\hat{i}$ <br> (a) $-90$    (b) $-50$    (c) 50    90 |
| **24-** | $(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta =$ ............... <br> (a) zero    (b) $\sin\theta$    1    (d) $\cos\theta$ |
| **25-** | If $\overrightarrow{A} = (k, 2)$ , $\overrightarrow{B} = 2i - j$ , and $\overrightarrow{A} \perp \overrightarrow{B}$ , then $k =$ ............... <br> 1    (b) $-1$    (c) $\pm 1$    (d) zero |
| **26-** | The polar form of the vector $\overrightarrow{A} = -3\,j$ is ............... <br> (a) $\left(-3, \frac{\pi}{2}\right)$    (b) $\left(3, \frac{\pi}{2}\right)$    (c) $\left(-3, \frac{3\pi}{2}\right)$    $\left(3, \frac{3\pi}{2}\right)$ |
| **27-** | If $\lVert 2k\overrightarrow{A} \rVert = \lVert -2\overrightarrow{A} \rVert$ where $\overrightarrow{A} \neq \hat{0}$ , then value of $k =$ ............... <br> (a) 1    (b) $-1$    $\pm 1$    (d) zero |
| **28-** | If the matrix A of order $2 \times 3$ and the matrix B of order $3 \times 3$ , then the matrix AB of order ............... <br> (a) $2 \times 2$    (b) $3 \times 3$    (c) $3 \times 2$    $2 \times 3$ |
| **29-** | If $\tan\theta = 3$ , then $\sec^2\theta =$ ............... <br> (a) 9    10    (c) $-10$    (d) 0.9 |
| **30-** | $3\tan\theta\cot\theta + 2\sin\theta\csc\theta + \cos\theta\sec\theta =$ ............... <br> (a) 1    (b) 3    (c) 5    6 |
| **31-** | If X is a matrix where $X \times \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ , then $X =$ ............... <br> (a) $\begin{pmatrix} 1 & 1 \end{pmatrix}$    (b) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$    I    (d) (1) |

**32-** By using determinants , then the area of triangle ABC in which :
$A(-4, -2)$ , $B(0, 3)$ , $C(0, 0)$ equal ..............

(a) $-6$ (b) $12$ (c) $-12$ $6$

**33-** If $\overrightarrow{A} = \left(6\sqrt{2}, \frac{3\pi}{4}\right)$ position vector of the point A , then A = ..............

(a) $(6, -6)$ $(-6, 6)$ (c) $(6, 6)$ (d) $(-6, -6)$

**34-** If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a skew matrix , then b + c = ..............

(a) 2 a (b) 2 d (c) 2 0

**35-** ABCD is parallelogram in which : $A = (7, -2)$ , $B = (15, 4)$ , $C = (9, 6)$
, then the coordinates of the point D = ..............

$(1, 0)$ (b) $(0, 1)$ (c) $(-1, 0)$ (d) $(0, -1)$

**36-** The point of intersection of medians (concurrent) to the $\Delta ABC$ in which
$A = (3, 2)$ , $B = (1, -2)$ , $C = (-1, 6)$ is ..............

(a) $(-1, 2)$ $(1, 2)$ (c) $(1, -2)$ (d) $(-1, -2)$

**37-** If O is the zero matrix of order $2 \times 2$ , then the number of its elements = ..............

(a) zero (b) $\varnothing$ (c) 2 4

**38-** The solution set of the equation $\sqrt{3}\tan\theta = 1$ , $90° < \theta < 270°$ is ..............

(a) $\{30°\}$ (b) $\{150°\}$ $\{210°\}$ (d) $\{240°\}$

**39-** If $A = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ , $B = (2 \quad 3)$ , then BA = ..............

$(-4)$ (b) $(4)$ (c) $\begin{pmatrix} 2 & 3 \\ -4 & -6 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & -4 \\ 3 & -6 \end{pmatrix}$

**40-** If $B^t A^t = \begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}$, then $(AB)^t =$ ..............

$\begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -3 \\ 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} -3 & 2 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix}$

**41-** If $\sin^2\theta + \cos^2\theta + \tan^2\theta =$ ..............

(a) 1 $\sec^2\theta$ (c) $\cot^2\theta$ (d) $\tan^2\theta$

**42-** In $\Delta ABC$, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} =$ ..............

(a) O $\overrightarrow{0}$ (c) zero (d) $\overrightarrow{A}$

**43-** If $\overrightarrow{A} = (\ell, -3)$, $\overrightarrow{B} = (2, -2)$, $\overrightarrow{A} \perp \overrightarrow{B}$, then $\ell =$ ..............

(a) 3 $-3$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

**44-** If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a skew matrix, then $b + c =$ ..............

(a) 2 a (b) 2 d (c) 2 0

**45-** $(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta =$ ..............

(a) zero (b) $\sin\theta$ 1 (d) $\cos\theta$

**46-** If $\overrightarrow{v_A} = 70\,\hat{i}$, $\overrightarrow{v_B} = -20\,\hat{i}$, then $\overrightarrow{v_{AB}} =$ .............. $\hat{i}$

(a) $-90$ (b) $-50$ (c) 50 90

**47-** From the top of a tower, its height is 80 m., an observer measures the depression angle of a car lies on the same plane of the tower base and it was $32^\circ\ 18'$, then the distance between the car and the tower base equals ..............

(a) 50.6 m. (b) 42.7 m. 126.5 m. (d) 149.7 m.

# كيفية طباعة صفحات معينة من ملف معين

## مثلا ازاى نطبع الصفحات من صفحة 4 الى صفحة 9